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CONCERNING THE DEFINITIONS OF THE MEAN ABSORPTION COEFFICIENT

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THE ANALYSIS of radiation transfer problems in absorbing and emitting media is complicated by the fact that the radiative properties are dependent on wavelength. The first approximation of a physical problem is obtained by treating the radiating matter as gray and introducing an appropriate mean absorption coefficient. This, of course, is a mathematical idealization of the physical situation. However, the gray case is of particular interest since it provides a physically significant standard of comparison for interpreting the more general case and simplifies considerably a difficult problem. Therefore, the situation in which the spectral absorption coefficient κ_λ* is supposed to be independent of the wavelength by replacing it with a mean absorption coefficient integrated over the entire spectrum is worthy of careful analysis.

There is a lack of agreement among the astrophysicists [1] on the most appropriate definition for the mean absorption coefficient. They attempt to define, for example, the mean in such a manner that the condition of radiative equilibrium, i.e. constant radiant energy flux, is maintained in a stellar atmosphere. The definitions that result from this approach may be highly successful when applied to the study of radiation transfer in the atmospheres of stars and planets; however, they are not meaningful in the more general engineering problem when energy transport by conduction and/or convection is also present. Investigators who have been concerned with radiation transfer problems in systems of engineering nature, where surfaces are of necessity present, have used either the Rosseland [2, 3] or the Planck mean absorption coefficients [3, 4].

For radiation in strongly absorbing media when the radiant energy flux can be approximated by a simple diffusion equation, the Rosseland mean absorption coefficient should be used. However, whenever the radiation transfer problem is formulated in terms of the integral equations the use of Planck's mean absorption coefficient might not be appropriate in some instances. In this note we discuss a more logical basis for defining the mean absorption coefficient, which should be valid for problems of engineering nature, that is, in situations in which energy transport is by radiation alone or when radiation interacts with other modes of energy transport.

* The subscript λ refers to a particular wavelength.

Consider a homogeneous medium which is capable of emitting and absorbing thermal radiation of wavelength λ. Assume that the medium is in local thermodynamic equilibrium and that the index of refraction is n_λ. The temperature may vary from point to point in the medium, but each point may be characterized by a definite temperature T, so that the matter at each point is behaving as if in local thermodynamic equilibrium. If I_λ is the spectral intensity of radiation in a given direction s then the conservation equation of a monochromatic pencil of radiation, or the equation of transfer, can be expressed as†

$$(\mathbf{s}\cdot\nabla) I_\lambda = \kappa_\lambda n_\lambda^2 I_{b,\lambda} - \kappa_\lambda I_\lambda, \qquad (1)$$

where $I_{b,\lambda}$ is the black body intensity of radiation *in vacuo* given by Planck's function. The first term on the right-hand side of equation (1) accounts for emission and the second for absorption of radiation. Integration of equation (1) over all solid angles ($\Omega = 4\pi$) yields the conservation equation of monochromatic radiant energy:

$$\nabla\cdot\mathscr{F}_\lambda = 4\pi\kappa_\lambda n_\lambda^2 I_{b,\lambda} - \kappa_\lambda \mathscr{G}_\lambda, \qquad (2)$$

where the radiant energy flux vector $\mathscr{F}_\lambda$ is defined as

$$\mathscr{F}_\lambda = \int_{\Omega = 4\pi} I_\lambda \mathbf{s}\, d\Omega \qquad (3)$$

and the radiant energy incident on the boundary of an element of volume of radiating media from all directions is denoted by

$$\mathscr{G}_\lambda = \int_{\Omega = 4\pi} I_\lambda\, d\Omega. \qquad (4)$$

The absorption coefficient κ_λ is related to the commonly employed mass absorption coefficient κ_λ' by the relation

$$\kappa_\lambda = \rho\kappa_\lambda', \qquad (5)$$

where ρ is the density of radiating media.

The physical meaning of the right-hand side of equation (2) can be clarified when we note that the term $4\pi n_\lambda^2 I_{b,\lambda}$ is the product of the spectral radiant energy density of a black body at the local molecular temperature times the

† Reference 1, pp 6–17.

local velocity of light c. While $\mathscr{G}_\lambda$ is related to the local radiant energy density of space u_λ* by $\mathscr{G}_\lambda = c\, u_\lambda$.

If the co-ordinate s is laid off along the direction **s**, the directional derivative $(\mathbf{s} \cdot \nabla)\, I_\lambda$ becomes $\mathrm{d}I_\lambda/\mathrm{d}s$, and the formal solution of equation (1) can readily be written as

$$I_\lambda(s) = I_\lambda(0) \exp\left[-\int_0^s \kappa_\lambda(s')\,\mathrm{d}s'\right] + \int_0^s n_\lambda^2(s')\,\kappa_\lambda(s')\,I_{b,\lambda}(s') \exp\left[-\int_s^{s'} \kappa_\lambda(s'')\,\mathrm{d}s''\right]\mathrm{d}s'. \quad (6)$$

In writing (6) it was assumed that the intensity of radiation leaving the surface of the system at $s = 0$ is given by $I_\lambda(s) = I_\lambda(0)$. It should be noted that if the surfaces of the system are real the spectral distribution $I_\lambda(0)$ leaving the surface will not correspond to black body radiation.

Integration of the conservation equation of monochromatic radiant energy (2) over the entire spectrum yields

$$\nabla \cdot \int_0^\infty \mathscr{F}_\lambda\,\mathrm{d}\lambda = 4\pi \int_0^\infty \kappa_\lambda\, n_\lambda^2\, I_{b,\lambda}\,\mathrm{d}\lambda - \int_0^\infty \kappa_\lambda\, \mathscr{G}_\lambda\,\mathrm{d}\lambda. \quad (7)$$

Defining the total (integrated over all wavelengths) radiant energy flux vector as

$$\mathscr{F} = \int_0^\infty \mathscr{F}_\lambda\,\mathrm{d}\lambda \quad (8)$$

and introducing the "mean emission coefficient",† corresponding to the emission or radiation, $\bar{\kappa}_e$, as

$$\bar{\kappa}_e = \frac{\int_0^\infty \kappa_\lambda\, n_\lambda^2\, I_{b,\lambda}\,\mathrm{d}\lambda}{\int_0^\infty n_\lambda^2\, I_{b,\lambda}\,\mathrm{d}\lambda} = \frac{\int_0^\infty \kappa_\lambda\, n_\lambda^2\, E_{b,\lambda}\,\mathrm{d}\lambda}{\int_0^\infty n_\lambda^2\, E_{b,\lambda}\,\mathrm{d}\lambda} = \frac{\int_0^\infty \kappa_\lambda\, n_\lambda^2\, E_{b,\lambda}\,\mathrm{d}\lambda}{n^2\, E_b} \quad (9)$$

as well as the "mean absorption coefficient" corresponding to absorption of radiation, $\bar{\kappa}_a$, as

$$\bar{\kappa}_a = \frac{\int_0^\infty \kappa_\lambda\, \mathscr{G}_\lambda\,\mathrm{d}\lambda}{\int_0^\infty \mathscr{G}_\lambda\,\mathrm{d}\lambda} = \frac{\int_0^\infty \kappa_\lambda\, \mathscr{G}_\lambda\,\mathrm{d}\lambda}{\mathscr{G}}, \quad (10)$$

equation (7) can be written as

$$\nabla \cdot \mathscr{F} = 4\bar{\kappa}_e\, n^2\, E_b - \bar{\kappa}_a\, \mathscr{G}. \quad (11)$$

* Reference 1, p. 13.

† The definition of the emission coefficient j_λ as employed by the astrophysicists is different from that used here. They define a spectral emission coefficient as $j_\lambda = \kappa_\lambda\, n_\lambda^2\, I_{b,\lambda}$.

Since the first term on the right-hand side of (7) represents the radiant energy emitted from an element of volume per unit time in all directions, the weight function $\pi n_\lambda^2\, I_{b,\lambda}$ $(= n_\lambda^2\, E_{b,\lambda})$ should be used in defining the mean emission coefficient $\bar{\kappa}_e$. This is correct physically because the amount of radiant energy emitted by the radiating matter should depend on the local conditions only. Note that for the case when the index of refraction of radiating matter is unity, $\bar{\kappa}_e$ reduces to the definition of Planck's mean absorption [1]. The second term on the right-hand side of (7) represents the amount of radiant energy absorbed by the matter. The mean absorption coefficient $\bar{\kappa}_a$, therefore, depends not only on the physical nature and temperature of the radiating medium but also on the spectral distribution of the incident radiation, $\mathscr{G}_\lambda$. The spectral characteristics of $\mathscr{G}_\lambda$ will depend not only on the spectral characteristics of the radiating media but also on those of the surfaces [see equation (6)]. Therefore the spectral distribution of $n_\lambda^2\, E_{b,\lambda}$ will not be the same as that of $\mathscr{G}_\lambda$, and in general $\bar{\kappa}_e \neq \bar{\kappa}_a$. If and only if κ_λ is independent of λ or if $n_\lambda^2\, E_{b,\lambda} = \mathscr{G}_\lambda$ will $\bar{\kappa}_e = \bar{\kappa}_a$.

It should be noted that a parallel distinction between emission and absorption of radiation is made in a more familiar case of radiant heat transfer from surfaces [5]. The definition of the total emissivity and the total absorptivity are similar to those of the mean emission and the mean absorption coefficients, respectively.

For completeness sake, we indicate here the definition of the Rosseland mean absorption coefficient [6],

$$\frac{1}{\kappa_R} = \int_0^\infty \left(\frac{1}{\kappa_\lambda}\right) \frac{\mathrm{d}(n_\lambda^2\, E_{b,\lambda})}{\mathrm{d}T}\,\mathrm{d}\lambda \Bigg/ \int_0^\infty \frac{\mathrm{d}(n_\lambda^2\, E_{b,\lambda})}{\mathrm{d}T}\,\mathrm{d}\lambda, \quad (12)$$

where T is the absolute temperature. At distances far (optically) away from the boundaries and a system close to equilibrium such that the temperature and the radiative properties do not change much within one photon mean free path, the radiant energy flux vector can be approximated by a simple diffusion type equation,

$$\mathscr{F} = -\frac{1}{3\kappa_R} \nabla\, (4n^2\, E_b). \quad (13)$$

The term $4n^2\, E_b$ is the product of the total radiant energy density at the local temperature and the velocity of light.

For geometrically complex systems Hottel [7] has proposed to calculate the absorption coefficients from real-gas emissivity and absorptivity data in series form representative of a sum of a small number of "gray" gases. The mixed gray-gas concept allows for correct use of the mean absorption coefficient and is useful for zonal-type analysis of radiant heat transfer. The total absorptivity of a gas, however, depends not only on the temperatures of the gas and the enclosing surfaces but also on the mean beam length, i.e. the shape of the experimental apparatus. Thus the local variation of the mean absorption coefficient cannot be determined with confidence from this type of data in situations where large temperature gradients are expected to occur.

Unfortunately, $\bar{\kappa}_e$ cannot be calculated unless the temperature distribution is known, so that $E_{b,\lambda}$ can be predicted at any point in the medium. The index of refraction n_λ is generally a weak function of wavelength and can as a first approximation be assumed to be constant. The value of $\bar{\kappa}_a$ cannot be calculated unless the nature of the incident radiation $\mathcal{G}_\lambda$ is clearly specified. Usually, both $E_{b,\lambda}$ and $\mathcal{G}_\lambda$ are not known rigorously until the whole problem has been solved; however, in many problems reasonable first approximations can be found for these quantities. Both $\bar{\kappa}_e$ and $\bar{\kappa}_a$ can then be corrected as more accurate information on $E_{b,\lambda}$ and $\mathcal{G}_\lambda$ becomes available.

The arbitrariness associated with the definition of a mean absorption coefficient has been eliminated by introducing a "mean emission coefficient" $\bar{\kappa}_e$ and a "mean absorption coefficient" $\bar{\kappa}_a$. The physical meanings of $\bar{\kappa}_e$ and $\bar{\kappa}_a$ are clear. It is believed that the distinction between $\bar{\kappa}_e$ and $\bar{\kappa}_a$ gives a more logical and physically realistic basis for their use in the integrated (over all wavelengths) equation for conservation of radiant energy and in the calculation of radiant heat transfer. However, a word of caution should be interjected here. The radiating media is likely to be optically thin for some important spectral regions and optically thick for others. Thus, in order to obtain accurate results, it will probably be necessary to devise different approximation schemes for the different spectral regions for the calculation of the radiant heat flux.

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